NO.
DATE / /

1. Fill in the following blanks with the correct numbers

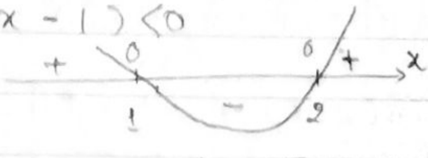
(1) when $a>0$, then what is the range of x that satisfies the following inequality : $ax^2 - 3ax + 2a < 0$

So we've $ax^2 - 3ax + 2a < 0$

$a(x^2 - 3x + 2) < 0$

$a(x-2)(x-1) < 0$

So $a > 0$,



$\forall x \in]1, 2[\quad \therefore S =]1, 2[$

(2). If $4^{3x-1} - 2^{5x-4} = 0$, then $x = \boxed{}$

we've $4^{3x-1} - 2^{5x-4} = 0$

$(2^2)^{3x-1} = 2^{5x-4}$

$2^{6x-2} = 2^{5x-4} \to 6x - 2 = 5x - 4 \to x = -2$

$\boxed{x = -2}$

(3) $10^{\log_{10} 5} = 5$ So $a^{\log_a b} = b$, $\begin{cases} a>0 ; a \neq 1 \\ b>0 \end{cases}$

(4) when α and β are the solutions of the quadratic equation $x^2 - 5x + 3 = 0$ then $\alpha^2 + \beta^2 = \boxed{}$, $(\alpha - \beta)^2 = \boxed{}$

we've $x^2 - 5x + 3 = 0$

$(x - \alpha)(x - \beta) = 0 \to \begin{cases} \alpha + \beta = 5 & (1) \\ \alpha\beta = 3 & (2) \end{cases}$

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

notice at $\alpha + \beta = 5 \to (\alpha+\beta)^2 = 5^2 \to \alpha^2 + \beta^2 + 2\alpha\beta = 25$

$\to \alpha^2 + \beta^2 + 6 = 25 \to \alpha^2 + \beta^2 = 19$ ans 1

So $\alpha^2 + \beta^2 = 19 \to \alpha^2 - 2\alpha\beta + \beta^2 = 19 - 2(3) \to (\alpha - \beta)^2 = 13$ ans 2

(5) When $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{a} - \vec{b}| = \sqrt{7}$, then the degree measere of the angle between $\vec{a}$ and $\vec{b}$ is $\boxed{}°$

So we've a formula : $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\vec{a}, \vec{b})$ (1)

we've $|\vec{a} - \vec{b}| = \sqrt{7} \rightarrow (|\vec{a} - \vec{b}|)^2 = (\sqrt{7})^2 \rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\vec{b} = 7$

$2\vec{a}\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 7 \rightarrow 2\vec{a}\vec{b} = -2 \rightarrow \vec{a}\vec{b} = -1$

(1), $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\vec{a}, \vec{b}) \rightarrow -1 = 2\cos(\vec{a}, \vec{b}) \rightarrow \cos(\vec{a}, \vec{b}) = -\frac{1}{2}$

So $0° \leq (\vec{a}, \vec{b}) \leq 180°$ then $(\vec{a}, \vec{b}) = 120°$

(6) when ΔABC is a triangle where $\angle A = 30°$, then $\sin(\angle B + \angle C)$ is $\boxed{}$

So we've $\angle A + \angle B + \angle C = 180°$

then $\sin(\angle B + \angle C) = \sin(180° - \angle A) = \sin(\angle A)$

$\sin(\angle B + \angle C) = \sin(30)° = \frac{1}{2}$

$\therefore \sin(\angle B + \angle C) = 1/2$ ans

(7) How many multiples of 3 are there among integers from 100 to 200?

We've $100 < 3n < 200 \quad n \in \mathbb{N}$

Consider ~~$n = \lfloor \frac{100}{3} \rfloor = 33 \rightarrow$ first n~~ $n = \lceil \frac{100}{3} \rceil = 34 \rightarrow$ first n

~~$n = \lceil \frac{200}{3} \rceil =$~~ $n = \lfloor \frac{200}{3} \rfloor = 66 \rightarrow$ last n

therefore multiplies of 3 among integers from 100 to 200

there're 33 ans

(8) When x^3+ax^2+bx+5 is dividosible by $x-1$ and has a remainder of 5 when divided by $x-2$, then $a=\square$, $b=\square$

first condition: $x^3+ax^2+bx+5=(x-1)Q(x)+0$

when $x=1$, $1+a+b+5=6$

$a+b=-6$ (1)

Second condition: $x^3+ax^2+bx+5=(x-2)Q(x)+5$

when $x=2$, $8+4a+2b+5=5$

$4+2a+b=0$

$2a+b=-4$ (2)

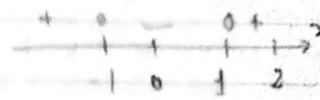
(2) - (1) sides by sides, $(2a+b)-(a+b)=-4-(-6)$

$a=2$ then $b=-8$

$\therefore$ $a=2$ ans ① and $b=-8$ ans ②

(9) let $f(x)=|x^2-1|$. Then $f(0)=\square$, $\int_0^2 f(x)\,dx=\square$

So $f(0)=|0^2-1|=1 \rightarrow f(0)=1$ ans ①

$\int_0^2 f(x)\,dx=\int_0^2 |x^2-1|\,dx$ So  for $y=x^2-1$

$=\int_0^1 |x^2-1|\,dx+\int_1^2 |x^2-1|\,dx=\int_1^0 (x^2-1)\,dx+\int_1^2 (x^2-1)\,dx$

$=\left(\frac{x^3}{3}-x\right)\Big|_1^0+\left(\frac{x^3}{3}-x\right)\Big|_1^2=\left(1-\frac{1}{3}\right)+\left(\frac{8}{3}-2\right)-\left(\frac{1}{3}-1\right)$

$=2$

$\therefore$ $f(0)=1$ and $\int_0^2 f(x)\,dx=2$

(10) assume that a, b and c are consecutive terms of arithmetic progression ($a<b<c$). if $a+b+c=24$ and $abc=440$, then

$a = \boxed{}$, $b = \boxed{}$, $c = \boxed{}$

a, b, c are consecutive terms of arithmetic progression

$\begin{cases} a+b+c=24 \\ abc=440 \end{cases}$ assume $a=a_1$ then $\begin{cases} a=a_1 \\ b=a_1+d \\ c=a_1+2d \end{cases}$ (d - distance)

$\rightarrow \begin{cases} a_1+a_1+d+a_1+2d=24 \\ a_1(a_1+d)(a_1+2d)=440 \end{cases} \rightarrow \begin{cases} 3a_1+3d=24 \\ a_1(a_1+d)(a_1+2d)=440 \end{cases}$

$\rightarrow \begin{cases} a_1+d=8 \\ a_1(8)(8+d)=440 \end{cases} \rightarrow \begin{cases} a_1+d=8 \\ a_1(8+d)=55 \end{cases}$

$\rightarrow \begin{cases} d=8-a_1 \\ 8a_1+a_1(8-a_1)=55 \end{cases} \rightarrow \begin{cases} d=8-a_1 \\ 16a_1-a_1^2=55 \end{cases} \rightarrow \begin{cases} d=8-a_1 \\ a_1^2-16a_1+55=0 \end{cases}$

$\rightarrow \begin{cases} d=8-a_1 \\ (a_1-5)(a_1-11)=0 \end{cases} \rightarrow \begin{cases} a_1=5 \text{ then } d=3 \\ a_1=11 \text{ then } d=-3 \end{cases}$

So $a<b<c$ then $a_1=5$ and $d=3$

$\therefore$ $a=5$, $b=8$, $c=11$

2. On the plane xy, there are 4 points, $O(0,0)$, $A(0,3)$, $B(0,-3)$, $C(4,0)$. Fill in the following blanks with the correct numbers

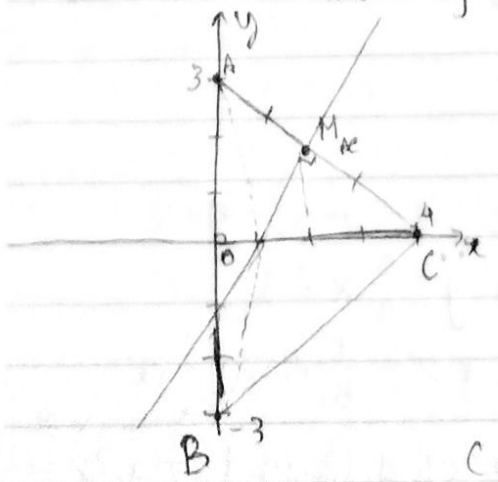
(1) The equation of the straight line AC is $\square x + \square y - \square = 0$

So $A(0,3)$ and $C(4,0)$ so $y = mx + c$: (AC)

When $x = 0$ and $y = 3$, $c = 3$
when $x = 4$ and $y = 0$, $m = -\frac{3}{4}$ $\rightarrow$ (AC): $y = -\frac{3}{4}x + 3$

(AC): $3x + 4y - 12 = 0$ $\rightarrow$ ① $= 3$, ② $= 4$, ③ $= 12$

(2) The Coordinates of the Circumcenter of ΔABC are $\left(\frac{\square}{8}, \square\right)$



we've $A(0,3)$ and $C(4,0)$

$M_{AC}\left(\frac{4}{2}, \frac{3}{2}\right) = M_{AC}\left(2, \frac{3}{2}\right)$

$(d\perp_{AC})$: $y = mx + c$

So $m\left(-\frac{3}{4}\right) = -1 \rightarrow m = \frac{4}{3}$

$(d\perp_{AC})$: $y = \frac{4}{3}x + c$ through M_{AC}

$\frac{3}{2} = \frac{4}{3}(2) + c \rightarrow c = \frac{3}{2} - \frac{8}{3} = \frac{9-16}{6}$

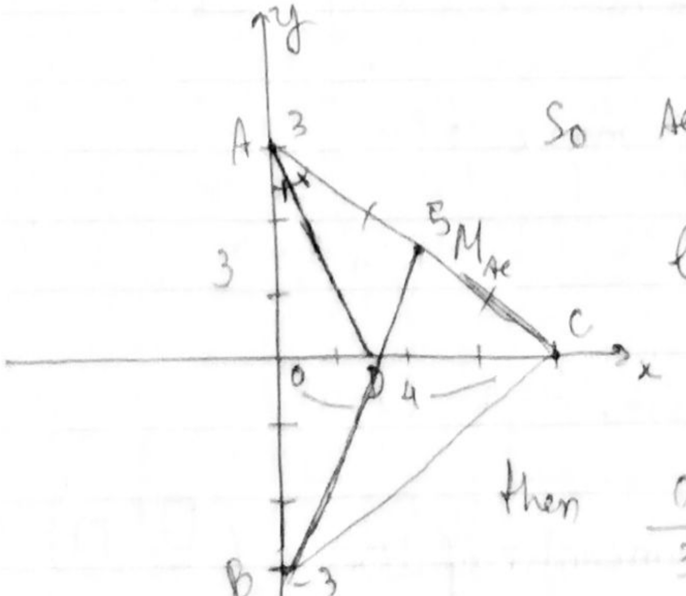
$c = -7/6 \rightarrow (d\perp_{AC})$: $y = \frac{4}{3}x - \frac{7}{6}$

and (OC): $\cancel{y \neq \emptyset x = 0}$ $y = 0$

so support $(d\perp_{AC}) = (OC)$, $\frac{4}{3}x - \frac{7}{6} = 0 \rightarrow \frac{4}{3}x = \frac{7}{6} \rightarrow x = \frac{7}{8}$

then $y = 0$ $\therefore$ ① $= 7$ and ② $= 0$

(3) When point D is the intersection of bisector of $\angle ABC$ and x-axis, then OD : DC = ① : ② and the coordinates of inner center of ΔABC are $\left(\frac{③}{2}, ④\right)$



So AO = 3 and AC = 5

let OD = a then DC = 4 - a

we've the condition that bisector of $\angle ABC$

then $\frac{a}{3} = \frac{4-a}{5} \rightarrow 5a = 12 - 3a$

$8a = 12 \rightarrow a = \frac{3}{2}$

then $OD = \frac{3}{2}$ and $DC = \frac{5}{2}$

then OD : DC = 3 : 5 ① = 3 and ② = 5

So (OC): $y = 0$ and (BM): $y = \frac{9}{4}x - 3$

support (OC) = (BM), $0 = \frac{9}{4}x - 3 \rightarrow x = \frac{4}{3}$ then $y = 0$

the Coordinates of the inner Center of ΔABC are $\left(\frac{\frac{8}{3}}{2}, 0\right)$

③ $= \frac{8}{3}$ and ④ = 0

3. The line (a): $y = x + k$ (k is a constant) is tangent to both the parabola (b): $y = x^2 - 5x + 7$ and the parabola (c): $y = x^2 + 3x - 1$. Point P is the point of tangency of the line (a) and the parabola (b), Q is the point of tangency of the line (a) and the parabola (c) and point R is the intersection of the parabola (b) and the parabola (c).

Fill in the following blanks with the correct numbers

(1) The number $k = \boxed{}$

(2) The x-coordinate of the point P is $\boxed{①}$, the x-coordinate of the point Q is ② and the x-coordinate of the point R is ③

(3) The area surrounded by the line (a), the parabola (b) and the parabola (c) is $\boxed{}$

(1)
$$\begin{cases} y = x + k & : (a) \\ y = x^2 - 5x + 7 & : (b) \\ y = x^2 + 3x - 1 & : (c) \end{cases}$$

So (a) is tangent to both (b) and (c)

then (a) = (b) and (a) = (c) have a sol

$x + k = x^2 - 5x + 7$ and $x + k = x^2 + 3x - 1$

$x^2 - 6x + 7 - k = 0$ and $x^2 + 2x - (1 + k) = 0$

then $7 - k = 9$ and $-(1 + k) = 1$

$k = -2$ satisfied and $k = -2$ satisfied

$\therefore k = -2$

(2) The x-coordinate of the point P is ①

So P is the point of tangency of (a) ∩ (b)

(a) = (b): $x - 2 = x^2 - 5x + 7 \rightarrow x^2 - 6x + 9 = 0 \rightarrow x_{1,2} = 3 \rightarrow ① = 3$

Q is ②

So Q is the point of tangency of (a) ∩ (c)

(a) = (c): $x - 2 = x^2 + 3x - 1 \rightarrow x^2 + 2x + 1 = 0 \rightarrow x_{1,2} = -1 \rightarrow ② = -1$

~~R is ① intersection~~ R is ③

So R is the point of intersection of the parabola (b) and (c)

(b) = (c): $x^2 - 5x + 7 = x^2 + 3x - 1 \rightarrow 8x = 8 \rightarrow x = 1 \rightarrow ③ = 1$

Ans $k = -2$, ① = 3, ② = -1 and ③ = 1